1. There are 903 people such that no two of them have the same age. They sit around the table in a random order. Find the expected value of the number of people who are older than both of their neighbours.

2. The mass content of hydrogen in a crystal hydrate $Na_2HPO_4 \cdot nH_2O$ (n integer) is between 5.4% and 5.8%. Find n.

3. Below you see a list of statements about server technologies. Choose the correct statements.

   1) RAID (Redundant Array of Inexpensive Disks) is a storage system especially designed for servers and doesn't work anymore if one of the disks fails.
   2) Port scanning can be used to determine active services in a system because most of them are running on documented standard ports.
   3) HTML is the common markup language for structuring the layout of a website and determining how servers communicate with the client.
   4) HTTP is the only protocol for file transmission between the client and the server.

4. Solve the equation $\log_2(x) - \log_2(5) = 2 + \log_2(5)$.

5. How many integer numbers from 0 to 250 (inclusive) are multiples of exactly one of the three numbers 3, 4, 6?

6. How many boolean tuples (A, B, C, D) make the following expression true?

   $$B(A + \overline{C}D) + \overline{A + \overline{B}C} + D(A\overline{C} + B)$$

7. The letters a, b, c, and d are arbitrary symbols that represent four elements with consecutive atomic numbers in the periodic table. An atom of element c has one valence electron. N represents an atom of nitrogen. Choose the correct formula.

   $$1) \; a_3c_2 \qquad\qquad 2) \, b_3N_2 \qquad\qquad 3) \; da \qquad\qquad 4) \; d_3N_2$$
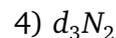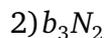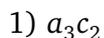
8. How many s-electrons are in a chromium atom in its ground electronic state? Give the integer number.

9. Physics students, who like fiddling, took 12 identical resistors with a resistance $R_i = 100\,\text{Ohm}$ each and connected them in the shape of a cube. They were wondering what the equivalent resistance of such a network would be if measured between two adjacent nodes. Help them by giving a numerical answer accurate to the first decimal.

   The answer should be given in the SI units unless otherwise stated. If the numerical result turns out to be an integer, it should be given as the answer. Otherwise round the result to the first decimal. $g = 10\,\text{m/s}^2$.

10. Find the last three digits of $2^{37993} - 1$.

11. What is the correct order of decreasing ionic radii?

    $$1) \; S^{2-} > Cl^- > Al^{3+} > Mg^{2+}$$
    $$2) \; Cl^- > S^{2-} > Al^{3+} > Mg^{2+}$$
    $$3) \; S^{2-} > Cl^- > Mg^{2+} > Al^{3+}$$
    $$4) \; Mg^{2+} > Al^{3+} > Cl^- > S^{2-}$$
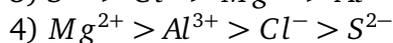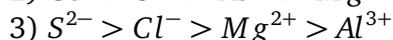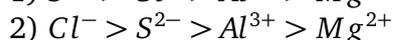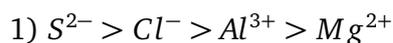
12. Below is the graph of the derivative $f'(x)$ of the function $f(x)$ defined on the interval $(-11, 11)$. How many local maximum points of the function $f(x)$ lie in the interval $[-10, 10]$?
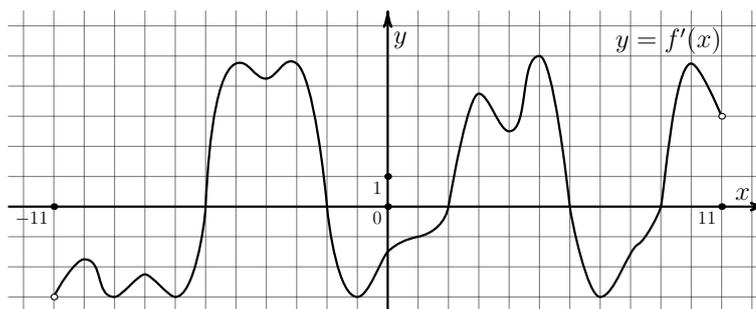
13. What is the minimal value of XY_{16} in base 10 when $12X4_5 = CY_{16}$?

14. A small mass is sliding without friction along the looped track shown in the Figure. It detaches from the loop at some point A and during the flight passes through the center O of the loop. What is the angle α (in degrees)?
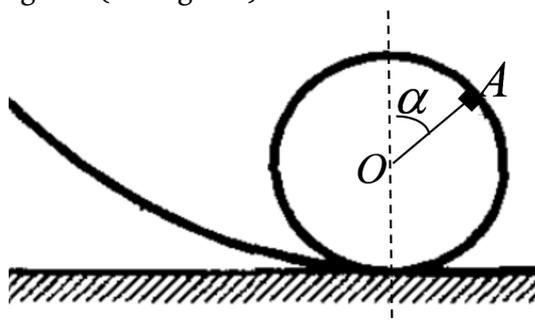
15. The organic compound contains 42.1% of oxygen, 47.4% of carbon, and 10.5% of hydrogen by mass. What is the total number of atoms in a molecule of this compound?

16. Calculate the resulting value of the following postfix expression:

$$3\ 4\ 6 + * 5\ 9\ 3\ / + *$$

17. What base N should be selected, so that the number $ABACABA_N$ converts to the smallest possible number in decimal system?

18. Suppose we have real numbers α and β such that $\alpha^3 - 3\alpha^2 + 5\alpha = 1$ and $\beta^3 - 3\beta^2 + 5\beta = 5$. Find the value of $\alpha + \beta$.

19. Internet Email Headers were originally limited to k-bit ASCII. What is the value of k?

20. How many times does the string "Divisible by 100!" appear?

```
1   #include <stdio.h>
2   int main() {
3     unsigned int count = 0;
4     for (int i = 0; i <= 5000; i++) {
5       count = i % 256;
6       if (count % 100 == 0) {
7         printf("Divisible by 100!\n");
8       }
9     }
10    return 0;
11  }
```

21. There is an unlimited supply of chips of three colours: red, green and blue. Find the number of ways to arrange 9 chips in a circle such that each colour is present. Arrangements are considered identical if and only if they can be obtained from each other by a rotation.

22. Suppose that O is the center of a circle, P is a point on this circle, and M is the midpoint of OP. Let Q and R be points on the circle such that they lie on the same side of the line OP, and $\angle QMO = \angle RMP$. Find the length of QR if QM and RM have lengths of 1527 and 519, respectively.

23. What is the volume of a regular octahedron (a Platonic solid with eight triangular faces) with the edge length 3? Give the answer accurate to three decimal places.

24. Three balls of radius $2\sqrt{6}-3$ are placed on the table with each ball touching the other two. The fourth ball with the same radius is placed above them so that it touches the first three ones. What is the height of the center of the fourth ball above the table?

25. Mr. Bagas bought 654 red candies, 101 blue candies, and 256 green candies for his son. His son doesn't want all the candies, he just wants to take 119 candies of the same color. Mr. Bagas wants his son to take as much as possible, so he puts all of them in one bag and asks his son to take them one by one from the bag, until his son has 119 candies of some color. How many candy can Mr. Bagas's son take, at most?

26. When a point charge q is at a distance $3R$ from the center of a metal sphere of radius R the interaction force between the charge and the sphere is equal to zero. What is the charge of the sphere?

1) $\frac{15}{192}q$; 2) $\frac{17}{192}q$; 3) $\frac{5}{64}q$; 4) $\frac{7}{64}q$.

27. You lie on the beach. The sea is quiet and you can see the sunset. Moreover, you can observe this phenomenon twice: the first time, when you lie, and the second time, when you stand up. The time that elapsed between two sunsets was 11.1 s, and you are 1.7 m tall. Using this time and your height, calculate the radius of the Earth. Give the answer in millions of meters accurate to the first decimal.

The answer should be given in the SI units unless otherwise stated. If the numerical result turns out to be an integer, it should be given as the answer. Otherwise round the result to the first decimal. $g = 10 \text{ m/s}^2$.

28. A ramp with an inclination angle of $\alpha = 30°$ and a length of $L = 30$ m is placed on the horizontal ground. A body of negligible dimensions is pushed up the ramp at the initial velocity of $v_0 = 20$ m/s. After reaching the end of the ramp, the body continues to move through air until it lands on the ground. Determine the body speed when it hits the ground. The coefficient of friction between the body and the ramp is $\mu = \frac{\sqrt{3}}{15}$. Air resistance is negligible.

The answer should be given in the SI units unless otherwise stated. If the numerical result turns out to be an integer, it should be given as the answer. Otherwise round the result to the first decimal. $g = 10 \text{ m/s}^2$.

29. This problem originates from the collection written by Alcuin of York approximately at 800 CE. A buyer says: „I want to buy 100 pigs with 100 denarii; a boar is bought for 10 denarii, a sow for five denarii, and two piglets for one denarius." How many boars, sows, and piglets he should buy to spend all money? Give the number of sows as an answer.

30. 100 ml of 30% perchloric acid solution ($\rho = 1.11$ g/mL) and 300 ml of 20% sodium hydroxide solution ($\rho = 1.10$ g/mL) were mixed. How many milliliters of water should be added to this mixture, so that the mass fraction of sodium hydroxide in the resulting solution be 11%? Take $A_r(Cl) = 35.5$.

1) 11.2 2) 38.5 3) 123 4) 159

31. Determine the output of the following program:

```
1   #include<bits/stdc++.h>
2   using namespace std;
3   int k = 0;
4   vector<bool> t(17,true);
5   bool f(int l, int r) {
6     k++;
7     if (l == r) return t[l];
8     int mid = (l + r) / 2;
9     if ((int)log2(r - l + 1) % 2 == 1) {
10      return f(l, mid) || f(mid + 1, r);
11    } else {
12      return f(l, mid) && f(mid + 1, r);
13    }
14  }
15  int main() {
16    t[1] = false;
17    for (int i = 2; i < 17; i++) {
18      if (t[i]) {
19        for (int j = i * i; j < 17; j += i) {
20          t[j] = false;
21        }
22      }
23    }
24    bool result = f(1, 16);
25    cout << k << endl;
26  }
```

32. A triangle ABC is given. Let M be the point on the side AB such that $AM : MB = 1 : 2$. A line through M crosses the segment BC at an internal point Q and the ray CA at a point P. Find the largest possible value of the ratio S_{ABC}/S_{PQC}.

33. Which of the following compounds give solutions with pH > 7?

$\quad$ 1) HCl $\qquad$ 2) KCN $\qquad$ 3) NH_4Cl $\qquad$ 4) CH_3NH_2 $\qquad$ 5) CH_3OH

34. The maximum height a flea can reach by jumping vertically is $h = 0.8$ m. Find the maximum distance the flea would be able to cover by jumping at the same initial velocity.

The answer should be given in the SI units unless otherwise stated. If the numerical result turns out to be an integer, it should be given as the answer. Otherwise round the result to the first decimal. $g = 10$ m/s^2.

35. Find the number of ordered pairs (a, b) of positive integers a and b such that $\gcd(a, b) = 24$ and $\operatorname{lcm}(a, b) = 1440$.

36. Determine the output of the following program:

```
1   #include <stdio.h>
2   int foo(int m, int n) {
3     if ((n <= 1) || (m <= 1)) {
4       return 2;
5     }
6     else {
7       return foo(n - 1, m) + foo(n, m - 1);
8     }
9   }
10  int main() {
11    printf("%d", foo(5, 4));
12  }
```

37. To what exact depth h must a thin-walled glass with its bottom facing up be immersed, so that it will keep sinking by itself? The glass has a mass of $m = 0.1$ kg and a volume of $V = 0.0002$ m^3. The water density is $\rho = 1000$ kg/m^3 and the Earth gravitational acceleration is $g = 10$ m/s^2. Assume the volume of the glass itself to be negligible and the air temperature to remain constant. The depth of immersion is defined as the difference between water levels in the glass and in the reservoir. Atmospheric pressure is 10^5 Pa.

   The answer should be given in the SI units unless otherwise stated. If the numerical result turns out to be an integer, it should be given as the answer. Otherwise round the result to the first decimal. $g = 10$ m/s^2.

38. Chris has an unlimited number of 2-dollar, 7-dollar and 10-dollar coins. He wants to pay an amount of 48 dollars exactly. What is the minimum number of coins required?

39. The air pressure at sea level (0 km altitude) is about 1000 hPa (hectopascal). The higher you rise in the Earth's atmosphere, the lower is the air pressure: it decreases by 13% per kilometer. Choose the equation that describes the relation between the altitude x and the air pressure y.

   1) $y = 0.87 \cdot x$
   2) $y = 1000 \cdot 0.87^x$
   3) $y = x^{1.13}$
   4) $y = 1000 \cdot 1.13^x$

40. a, b, c, d, e, f and g are boolean variables. Find all possible combinations of values of them, for which expression

   ```
   (a or d) and (a or f) and (not a or not d) and (b or d) and (not b or not g)

   and (c or not f) and (not c or not e) and (d or f) and (not d or not f)
   ```

   is true.

   Give the answer as a simple string of 0 and 1 for every variable from a to g. For example if a, b, c, d are false and e, f, g are true, the answer is 0000111.

41. How many typewriter pages of text can be stored on a 360KB floppy disk, if each character occupies 7 bits and each page is filled with 60 lines of 80 characters? Round the result up to the nearest integer.

42. Consider the following sequence: 5, 8, 4, 3, 8, 6, 2. The sequence is going to be sorted in ascending order, and bubble sort algorithm that compares two adjacent numbers from right to left is going to be used. How many swaps are performed during the algorithm? During the k-th round of data procession, assuming there are n elements, first we compare the $(n-1)$-th and n-th element and swap them if necessary, then $(n-2)$-th and $(n-1)$-th, etc., and at the end of the k-th round we compare and swap the k-th and the $(k+1)$-th element.

43. Determine the minimum value of $\dfrac{e^x}{2 - 2x + x^2}$ if x is a non-negative real number.

44. 1.23 mmol of a monoprotic acid were dissolved in water. 50 mL of a solution were obtained with pH $= 1.91$. The solution was further diluted by water to the volume of 1.000 L. What is the pH of the final solution?

$$\text{1) 1.91} \qquad \text{2) 2.56} \qquad \text{3) 2.95} \qquad \text{4) 3.21}$$

45. What average speed (in km/h) is reached by a regional train which departs from Cologne at 8.35 am and arrives punctually at 12.05 pm at Frankfurt am Main, 245 km away from Cologne.

The answer should be given in the SI units unless otherwise stated. If the numerical result turns out to be an integer, it should be given as the answer. Otherwise round the result to the first decimal. $g = 10$ m/s^2.

46. When a 2 μC charge is moved from a point A to a point B, the work done by an electric field is 50 μJ. What is the potential difference?

The answer should be given in the SI units unless otherwise stated. If the numerical result turns out to be an integer, it should be given as the answer. Otherwise round the result to the first decimal. $g = 10$ m/s^2.

47. Find the maximum value of the expression

$$3\sin(x + 0.3\pi) - 4\sin(0.8\pi + x),$$

where x is a real number.

48. The ionization constant of water at $70°C$ is $25 \cdot 10^{-14}$. The value of pH at this temperature is:

    1) 6.30   2) 6.70   3) 7.35   4) 8.05

49. A hydrocarbon C_nH_{2n+2} reacted with oxygen in the sufficient quantity. Before the reaction, the mixture temperature was 150 $°C$ and its volume was 200 cm^3. After the reaction, the volume of products was 240 cm^3 at the same temperature and pressure. Find n.

50. How many different paths are in the graph below which lead from A to T?
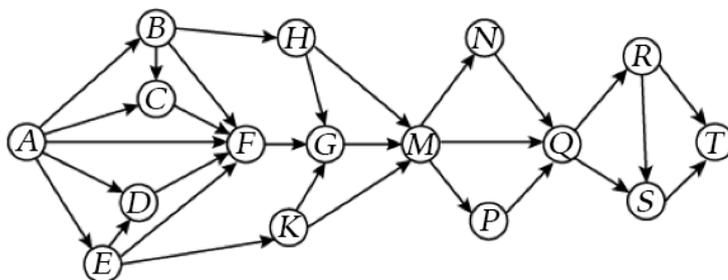
51. By how many times does the cross–sectional area S_2 of a vertical stream of water at a height $h = 5$ m differ from the cross-sectional area S_1 of the hose outlet? The speed of water at the outlet is $v_0 = 15.8$ m/s.

The answer should be given in the SI units unless otherwise stated. If the numerical result turns out to be an integer, it should be given as the answer. Otherwise round the result to the first decimal. $g = 10$ m/s^2.

52. Which of the following particles have (has) the sp^2 hybridization of a central atom?

    1) $BeCl_2$ 2) $Ni(CN)_4^{2-}$ 3) CO_3^{2-} 4) SF_6 5)BF_3

53. Which of the following mixtures is a buffer solution?

1) CH_3COOH (50 mL; 0.1 mol/L) + $NaOH$ (50 mL; 0.1 mol/L)
2) CH_3COOH (50 mL; 0.1 mol/L) + $NaOH$ (50 mL; 0.05 mol/L)
3) CH_3COOH (50 mL; 0.05 mol/L) + $NaOH$ (50 mL; 0.1 mol/L)
4) CH_3COOH (50 mL; 0.05 mol/L) + $NaOH$ (50 mL; 0.05 mol/L)

54. When 3.00 g of mineral M is treated with an excess acid, 0.4313 L of carbon dioxide escape (at 298 K and 1 bar). M contains 55.31% copper (by mass) and consists of four elements. Assume that all copper ions are in oxidation state +2. How many moles of oxygen are in one mole of M? Take $A_r(Cu) = 63.5$.

55. How many valid DFS subtrees, with root at the dark node (the root is the DFS algorithm starting point), are possible for the graph below? Note: A valid DFS subtree is the set of the edges which the DFS algorithm uses to traverse the graph.
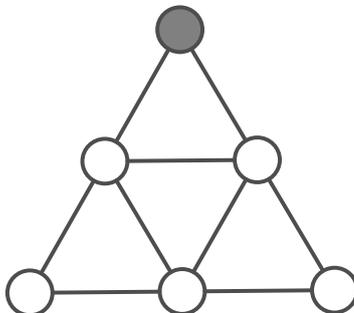
56. When 100 g of ice melts at $0°C$ in a room with $25°C$ the entropy change is 159 J/K. What would be the entropy change (in J/K) if this process occurs in a room with the temperature of $30°C$? Round up the result to integers (without units). The specific heat capacity of ice is $C_{ice} = 2.060 \, J/(g \cdot K)$ and that of liquid water is $C_{liq} = 4.184 \, J/(g \cdot K)$.

57. A ring of a radius $r = 0.5$ m is made of a wire of a diameter $d = 1$ mm and a resistivity of $\rho = 0.1$ Ohm·mm^2/m. The ring is placed in a varying uniform magnetic field which is perpendicular to the ring plane. The magnetic induction increases at a constant rate from zero to $B = 1$ T in $t_1 = 4$ s, then it decreases at a constant rate from B to $B/2$ in $t_2 = 1$ s. How much heat will be generated in the ring during the whole process? Give the answer in mJ.

The answer should be given in the SI units unless otherwise stated. If the numerical result turns out to be an integer, it should be given as the answer. Otherwise round the result to the first decimal. $g = 10$ m/s^2.

58. Which compound contains an element with the same oxidation number as chromium in $K_2Cr_2O_7$?

   1) Cl_2O_7 2) $K_3[Fe(CN)_6]$ 3) VO_2NO_3 4) K_2MnO_4 5) $H_2S_2O_3$

59. An unknown organic compound A consists of carbon, hydrogen, and oxygen. A 21.51 g sample of A contains $1.506 \cdot 10^{23}$ molecules. Hydrolysis of A in the presence of an acid gives the carboxylic acid B and the substance C. In addition, oxidation of C with potassium dichromate gives B. What is A?

1) ethyl acetate
2) methyl acrylate
3) ethyl butyrate
4) vinyl acetate
5) crotonic acid

60. What should be the value of p in the input in order to get 20 as the output?

```
1   def mystery(k, ar, p):
2     while k < len(ar) - 1:
3       if ar[k] > ar[k + 1]:
4         ar[k] = ar[k] + ar[k + 1]
5         ar[k + 1] = ar[k] - ar[k + 1]
6         ar[k] = ar[k] - ar[k + 1]
7         k = 0
8       else: k += 1
9     return a[p] + a[p - 1]
10  p = int(input())
11  ar = [15, 20, 2, 5, 18]
12  print(mystery(0, ar, p))
```

61. Find the number of sets $\{a, b, c\}$ of three distinct positive integers such that $a \cdot b \cdot c = 2310$.

62. A 0.2 kg ball is dropped from a height $h = 10$ m at an initial velocity $v = 10$ m/s. It bounces back to the initial height. How much energy was lost during the collision with the ground? (Neglect the air resistance).

    The answer should be given in the SI units unless otherwise stated. If the numerical result turns out to be an integer, it should be given as the answer. Otherwise round the result to the first decimal. $g = 10$ m/s^2.

63. Mary dropped a stick in a deep hole. After 10 s she heard the stick hit the bottom. If we assume no air resistance and the speed of sound to be 343 m/s, how deep was the hole?

    The answer should be given in the SI units unless otherwise stated. If the numerical result turns out to be an integer, it should be given as the answer. Otherwise round the result to the first decimal. $g = 10$ m/s^2.

64. The table below gives the values of standard Gibbs energy ΔG for a certain reaction at various temperatures. Choose all the correct statements about this reaction.

Temperature, K	298	350	400
ΔG, kJ/mol	40.4	30.0	20.0

    1) The standard entropy change of the reaction is negative.
    2) The standard entropy change of the reaction is positive.
    3) At higher temperatures, the reaction becomes less spontaneous.
    4) The standard enthalpy change of the reaction is positive.

65. A container A of a volume 100 cm^3 is filled with an ideal gas at a pressure of $12 \cdot 10^5$ Pa while a container B of a volume 400 cm^3 is empty. The containers A and B are connected by a tube of negligible volume with a tap T controlling a gas flow, it is initially closed. The temperature of the containers is maintained at 0°C by two separate water baths filled with melting ice. The tap T is then opened and an equilibrium state is reached. Peter heats the bath of container A until water boils. Find the net amount of gas, in millimoles, that passed through the connecting tube during the heating. (The gas constant is 8.31 J/(K·mol), the Avogadro constant is 6.02×10^{23} mol^{-1}, and the mass of a molecule of the ideal gas is $m = 4.52 \times 10^{-26}$ kg.)

    The answer should be given in the SI units unless otherwise stated. If the numerical result turns out to be an integer, it should be given as the answer. Otherwise round the result to the first decimal. $g = 10$ m/s^2.

66. Let O be the intersection of the diagonals of a convex quadrilateral $ABCD$. The following areas are known: $S_{ABO} = 1$, $S_{BCO} = 2$, $S_{CDO} = 3$. Find the area of the triangle DAO.

67. Find a three-digit number divisible by 9 and 11 such that after swapping the first and the last digits you would get 2/9 of the original number.

68. A cube of ice at $0°$C and water vapour at $100°$C have identical masses. The ice and the vapour are mixed in an insulated container. What is the final temperature of the equilibrium state (in $°$C)?

    The answer should be given in the SI units unless otherwise stated. If the numerical result turns out to be an integer, it should be given as the answer. Otherwise round the result to the first decimal. $g = 10$ m/s^2.

69. What is the two's complement of an 8-bit number ED (its complement with respect to 2^8). Give the answer in decimal.

70. When attempting to express binary XOR (exclusive or) only using binary NAND (not and), how many NAND elements will you need at least?

71. A solution of hydrochloric acid is added to 10 mL of a 0.2 M solution of sodium aluminate. The amount of precipitate produced when 10 mL of the HCl solution is added is equal to the amount of the precipitate when 20 mL of the HCl solution is added. What is the molar concentration (in M) of the HCl solution? Give the answer with 2 digits after decimal point and without units. Assume that only $Al(OH)_3$ precipitates.

72. A jet of water leaves a hose and hits a wall where its velocity becomes zero. If the hose cross-sectional area is 25 cm^2, the velocity of water is 50 m/s and the density of water is 1000 kg/m^3, what is the force acting on the wall (in Newtons)?

    The answer should be given in the SI units unless otherwise stated. If the numerical result turns out to be an integer, it should be given as the answer. Otherwise round the result to the first decimal. $g = 10$ m/s^2.

73. An airplane is about to land. In order to line up with a runway, it is making a turn with the engines shut down. The speed of the plane is $v = 300$ km/h, its mass is $m = 200,000$ kg. What is the radius of the turn, if the angle between the plane wings and the horizontal is $\alpha = 30°$? (Assume the plane to be flying along a horizontal circle and the force of lift be perpendicular to the wings.)

    The answer should be given in the SI units unless otherwise stated. If the numerical result turns out to be an integer, it should be given as the answer. Otherwise round the result to the first decimal. $g = 10$ m/s^2.

74. How many liters of the oxygen-ozone mixture containing 10 vol.% of ozone is needed to burn completely 42 liters of propane C_3H_8 at standard conditions? Give the integer answer without units.

75. In a nuclear power plant of 800 MW (Ostrovets city), uranium $^{235}_{92}U$ is used as the fuel. Fission of one nucleus releases an energy of 200 MeV. Determine a daily consumption of uranium (kg), if a power plant efficiency is $\eta = 20\%$. The Avogadro constant is $6.02 \cdot 10^{23}$ 1/mol and the elementary charge is $1.60 \cdot 10^{-19}$ C.

    The answer should be given in the SI units unless otherwise stated. If the numerical result turns out to be an integer, it should be given as the answer. Otherwise round the result to the first decimal. $g = 10$ m/s^2.

76. A flying shell has exploded into two fragments which flew apart at an angle $\alpha = 60°$. One fragment has a mass of $m_1 = 20$ kg and a velocity of $v_1 = 100$ m/s. The other one has a mass $m_2 = 80$ kg and a velocity of $v_2 = 25$ m/s. What is the energy (in kJ) released by the projectile explosion?

   The answer should be given in the SI units unless otherwise stated. If the numerical result turns out to be an integer, it should be given as the answer. Otherwise round the result to the first decimal. $g = 10$ m/s^2.

77. Find the value of the expression

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \ldots + \frac{1}{2017} - \frac{1}{2018} - \left(\frac{1}{1010} + \frac{1}{1011} + \ldots + \frac{1}{2018} \right).$$

   1) $-\frac{1}{2018}$ 　　　　　 2) 0 　　　　　 3) $\frac{1}{2018}$ 　　　　　 4) 1

78. Let F-restricted composition of number n be an ordered collection of positive Fibonacci numbers whose sum is n. Let $f(n)$ be the number of summands in all F-restricted compositions of number n. For example $f(3) = 8$, because $3 = 1 + 1 + 1 = 1 + 2 = 2 + 1 = 3$, and there are $3 + 2 + 2 + 1 = 8$ summands in total. You need to find $f(10)$.

79. The substance X decomposes at a rate of 5% per minute at 318 K and 15% per minute at 328 K. Decomposition is a first-order reaction. Determine the activation energy.

   1) 9.6 kJ/mol 　　　　 2) 86.7 kJ/mol 　　　　 3) 95.3 kJ/mol 　　　　 4) 100 kJ/mol

80. There is a weight on a smooth plane inclined at an angle of $\alpha = 30°$ to the horizon. At what horizontal acceleration should one move the plane to prevent the weight from sliding?

   The answer should be given in the SI units unless otherwise stated. If the numerical result turns out to be an integer, it should be given as the answer. Otherwise round the result to the first decimal. $g = 10$ m/s^2.